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MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2017/2018

EEM2036 – ENGINEERING MATHEMATICS III
(All Sections/Groups)

25 OCTOBER 2017
2.30 p.m. - 4.30 p.m.
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This exam paper consists of **7 printed pages** (including cover page and formula sheets) with **four questions** only.
2. Attempt **ALL questions**. The distribution of marks for each question is given.
3. Please write all your answers in the Answer Booklet provided. Show all relevant steps to obtain maximum marks.
4. Only NON-PROGRAMMABLE calculator is allowed.

Question 1

(a) Suppose that a matrix A is given as follows: $A = \begin{bmatrix} 2 & 3 \\ 0 & 3 \end{bmatrix}$

Find

- (i) the eigenvalues of A . (1 marks)
- (ii) the eigenvalues of A^{-1} . (2 marks)
- (iii) $\text{trace}(A^{-1})$. (2 marks)

(b) A system of non-homogeneous differential equations is given as follows:

$$\begin{aligned}x_1' &= -5x_1 + x_2 + 5 \\x_2' &= 4x_1 - 2x_2 + 50\end{aligned}$$

$\lambda_1 = -1$ and $\lambda_2 = -6$ are the eigenvalues of $A = \begin{bmatrix} -5 & 1 \\ 4 & -2 \end{bmatrix}$ while their

corresponding eigenvectors are $e_1 = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$ and $e_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ respectively.

- (i) Write down a matrix P that diagonalizes A . (2 marks)
- (ii) Find the inverse P^{-1} . (2 marks)
- (iii) Write down a diagonal matrix D such that $P^{-1}AP = D$. (2 marks)
- (iv) Hence, find the general solution to the system. (14 marks)

Question 2

(a) Suppose that $f(2.5) = \alpha$ and $f(3.5) = 3\alpha$, find α if the Composite Trapezoidal rule

with $n = 2$ gives $\int_2^4 f(x) dx = 4$ and with $n = 4$ gives $\int_2^4 f(x) dx = 16$. Calculate the error bound for $\int_2^4 f(x) dx$ for the case $n = 2$ if $f(x) = e^{2x}$. (15 marks)

Continued...

(b) Consider the following system of linear equations:

$$\begin{aligned}5x_1 - 3x_2 &= 10 \\ -x_1 + 10x_2 + 2x_3 &= 8 \\ -3x_2 - 6x_3 &= 5\end{aligned}$$

Set up the Jacobi iterative equations and perform the first two iterations starting with the initial approximation $(0,0,0)$. (10 marks)

Question 3

(a) Produce a double integral that gives the volume of the solid under the plane $x + 2y + 3z = 6$ and bounded by the coordinate planes at the first octant. Make a sketch of the solid. **Do not solve.** (5 marks)

(b) Suppose a double integral is given as follows: $\iint_R \frac{x+y}{(x^2+y^2)^{\frac{1}{2}}} dx dy$

in which R denotes the region of a circle of radius 1 bounded at the first quadrant.

- (i) Write down the transformation equations that convert the Cartesian coordinates into polar coordinates. (2 marks)
- (ii) Calculate the Jacobian based on part (i) transformation. (1 mark)
- (iii) State the intervals of integration of region R in polar coordinates. (1 mark)
- (iv) Hence, solve the double integral in polar coordinates. (4 marks)

(c) The base of a sand pile covers the region in the xy plane that is bounded by the parabola $y^2 + x = 4$ and the line $y = x$. The height of the sand is bounded above by $x^2 + y^2 + z^2 = 4$ and below by paraboloid $z = x^2 + y^2$. Express the volume of sand as triple integral and find the volume of sand. (12 marks)

Continued...

Question 4

- (a) Calculate the flux of F where F is given by

$$F = 3x^2 zi + 4y^2 xzj$$

across the surface $z = x^2 y + y^2 + 10$ with $0 \leq x \leq 2$, $1 \leq y \leq 4$ in the direction of the upper unit normal. (13 marks)

- (b) Suppose that the temperature T over a certain region in space is given by

$$T(x, y, z) = e^{-yz} + x^2 y + z^2 y$$

Find the rate change of the temperature at the point $O (2,7,1)$ in the direction $P (0,0,0)$ to $Q (1,2,-1)$. (12 marks)

Continued...

APPENDIX

TABLE OF FORMULAS

1. The
- n
- th Lagrange interpolating polynomial (LIP)

$$f(x) \approx P_n(x) = \sum_{i=0}^n L_i(x) f(x_i)$$

with

$$L_k(x) = \prod_{\substack{i=0 \\ i \neq k}}^n \frac{(x - x_i)}{(x_k - x_i)}.$$

2. Newton's divided-difference interpolating polynomial (NDDIP)

$$P_n(x) = f[x_0] + \sum_{k=1}^n f[x_0, x_1, \dots, x_k](x - x_0) \cdots (x - x_{k-1}).$$

3. The error in interpolating polynomial.

$$f(x) - P_n(x) = \frac{(x - x_0)(x - x_1) \cdots (x - x_n)}{(n+1)!} f^{(n+1)}(c_x)$$

for each $x \in [x_0, x_n]$, a number $c_x \in (x_0, x_n)$ exists.

4. Newton's forward-difference formula

$$P_n(x) = f[x_0] + \sum_{k=1}^n \binom{s}{k} \Delta^k f(x_0).$$

5. Newton's backward-difference formula

$$P_n(x) = f[x_n] + \sum_{k=1}^n (-1)^k \binom{-s}{k} \nabla^k f(x_n).$$

6. Forward difference formula

$$f'(x) \approx \frac{f(x+h) - f(x)}{h}.$$

Backward difference formula

$$f'(x) \approx \frac{f(x) - f(x-h)}{h}.$$

The error term for both forward and backward difference formula is

$$\left| \frac{h}{2} f''(c_x) \right|.$$

Continued...

7. Central difference formula

$$f'(x) \approx \frac{f(x+h) - f(x-h)}{2h}$$

with the error term

$$\left| \frac{h^2}{6} f^{(3)}(c_x) \right|$$

8. Trapezoidal rule

$$\int_a^b f(x) dx \approx \frac{h}{2} [f(a) + f(b)]$$

for some ξ in (a, b) and $h = b - a$, with the error term is $\left| \frac{h^3 f''(\xi)}{12} \right|$.

9. Composite Trapezoidal rule

$$\int_a^b f(x) dx \approx \frac{h}{2} \left[f(a) + f(b) + 2 \sum_{j=1}^{n-1} f(x_j) \right]$$

for some ξ in (a, b) and $h = \frac{b-a}{n}$, with the error term is $\left| \frac{(b-a)h^2 f''(\xi)}{12} \right|$.

10. Simpson's rule

$$\int_a^b f(x) dx \approx \frac{h}{3} \left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b) \right]$$

for some ξ in (a, b) and $h = \frac{b-a}{2}$, with the error term $\left| \frac{h^5}{90} f^{(4)}(\xi) \right|$.

11. Composite Simpson's rule

$$\int_a^b f(x) dx \approx \frac{h}{3} \left[f(a) + 2 \sum_{j=1}^{\left(\frac{n}{2}\right)-1} f(x_{2j}) + 4 \sum_{j=1}^{\frac{n}{2}} f(x_{2j-1}) + f(b) \right]$$

for some ξ in (a, b) and $h = \frac{b-a}{n}$, with the error term $\left| \frac{(b-a)h^4}{180} f^{(4)}(\xi) \right|$.

Continued ...

12. Newton-Raphson's method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad n = 0, 1, 2, \dots$$

13. Euler's method

$$y_{i+1} = y_i + hf(x_i, y_i)$$

with local error $\frac{h^2}{2} Y''(\xi_i)$ for some ξ_i in (x_i, x_{i+1}) .

14. Runge Kutta method of order two (Improved Euler method)

$$y_{i+1} = y_i + \frac{1}{2}(k_1 + k_2)$$

$$k_1 = hf(x_i, y_i)$$

$$k_2 = hf(x_i + h, y_i + k_1)$$

15. Runge Kutta method of order four

$$k_1 = hf(x_i, y_i),$$

$$k_2 = hf(x_i + \frac{1}{2}h, y_i + \frac{1}{2}k_1),$$

$$k_3 = hf(x_i + \frac{1}{2}h, y_i + \frac{1}{2}k_2),$$

$$k_4 = hf(x_{i+1}, y_i + k_3),$$

$$y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4).$$

End of Paper